A Research Proposal

on

Some dynamical problems in Magneto Micropolar Thermo Elasticity

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Introduction

"Magneto Micropolar Thermo Elasticity" is part of the mathematical theory of elasticity. This theory is concerned with interacting effects of an externally applied magnetic field on the elastic and thermo-elastic deformations of a solid body. If an electrically conducting elastic solid is subjected to a mechanical load while immersed in a varying magnetic field, the laws of Hooke and Maxwell will still determine the elastic field and electro-magnetic field respectively but superposition of these two fields results in by the fact that electro-magnetic field influences the elastic field by entering the elastic stress equations of motion as a body force while the elastic field in its turn influences the electro-magnetic field by modifying Ohm's law. The elastic properties are characterized mathematically by certain functional relationships connecting forces and deformations. In many applications of engineering a point wise description of the field quantities, such as stress, strain, temperature is required throughout the body and mathematical theory provides a consistent set of equations which can be solved to obtain all the above said quantities for given applied load and material.

The linear theory of elasticity is of paramount importance in the stress analysis of steel, which is the commonest material of construction. To a lesser extent, this theory also describes the behavior of some other materials such as concrete, wood and coal. Further this area of study has the possibilities of their extensive applications in geophysics, optics and acoustics. For example while discussing the propagation of seismic waves from earth's mantle to its core, Cagniard (1952), suggested that the existence of the earth's magnetic field may be taken into consideration for explaining certain phenomenon concerning these waves.

Classical theory of elasticity:

This theory is based upon Hooke's Law which was discovered in 1660 and published in 1676. This theory successfully explains the behavior of common materials of construction such as steel; aluminum and concrete provided the stresses do not cross the

elastic limits. In this theory it is assumed that the action across an infinitesimal surface element inside a material is equivalent to a force. Hooke's law for homogenous isotropic elastic solid is

$$t_{ij} = \lambda \delta_{ij} e_{mm} + 2\mu e_{ij}$$

Where λ and μ are Lame's constants, δ_{ij} is the kronecker delta.

Micropolar theory of elasticity:

In many cases classical theory fails to explain the discrepancies between the classical theory and experiments, especially when the microstructure of the body is significant, as in the case of neighborhood of the holes, notches and cracks, where the stress gradients are considerable. Generally, most of the natural and manmade materials possess a microstructure. Metals, polymers, composites, soils, rocks and concrete are typical media with microstructures. The term Micropolar was coined by A. C. Eringen (1966) and is used to explain deformation caused in elastic media with oriented particles. In this theory it is assumed that every material point possesses an internal structure. So, three displacement components (u_1, u_2, u_3) are used to represent the macroscopic motion of the material point, and three additional microrotational angles (ϕ_1, ϕ_2, ϕ_3) are used to characterize the rotation of microstructure within the material point. The interaction between two parts of a body is transmitted not only by a force but also by torque, resulting in asymmetric force stresses and couple stresses. This media represents the materials that are made up of dipole atoms or dumb-bell molecules and are subjected to surface and body couples. Micropolar materials are assigned an additional object called "director" to represent their orientation in comparison to classical continuum mechanics where a material particle is assigned a certain position irrespective of orientation, within a material body at any instant of time. So a micropolar continuum is a collection of interconnected particles which can undergo both translational as well as rotational motion. Following Eringen (1966) mathematical model for representing stress-strain relation in homogenous isotropic micropolar elastic media is taken as:

$$t_{ij} = \lambda u_{r,r} \delta_{ij} + \mu \left(u_{i,j} + u_{j,i} \right) + K \left(u_{j,i} - \varepsilon_{ijr} \phi_r \right)$$

$$m_{ij} = \alpha\phi_{r,r}\delta_{ij} + \beta\,\phi_{i,j} + \gamma\phi_j \,.$$

Micropolar thermoelasticity

It is required in all elasto-dynamic problems to consider the temperature's dependence of displacements thus giving rise to coupled thermoelastic equations. This theory consists of heat conduction equation and stress strain that are produced due to the flow of heat. It makes it possible to calculate the stresses produced by the temperature field and to determine the temperature distribution due to the action of time dependent forces and heat sources. The theory was developed by Eringen and Nowacki by extending the theory of micropolar media to include thermal effects and is known as micropolar coupled theory of thermoelasticity. Following Lord and Shulman (1967), Green and Lindsay (1972) and Eringen (1984), the mathematical model for calculating stress-strain together with modified Fourier law of heat conduction for homogenous isotropic micropolar generalized thermoelastic solid is:

$$t_{ij} = \lambda u_{r,r}\delta_{ij} + \mu\left(u_{i,j} + u_{j,i}\right) + K\left(u_{j,i} - \varepsilon_{ijr}\phi_r\right) - \nu\left(\mathrm{T} + \tau_1\frac{\partial T}{\partial t}\right)\delta_{\mathrm{ij}} \ ,$$

$$m_{ij} = \alpha\phi_{r,r}\delta_{ij} + \beta\,\phi_{i,j} + \gamma\phi_{j,i} \ ,$$

$$K^*\nabla^2 T = \rho C^*\left(\frac{\partial T}{\partial t} + \tau_0\frac{\partial^2 \mathrm{T}}{\partial t^2}\right) + \nu T_0\left(\frac{\partial}{\partial t} + \Xi\,\tau_0\frac{\partial^2}{\partial t^2}\right)u_{i,i}$$

Magneto Micropolar elasticity

This theory is concerned with the interacting effects of an externally applied magnetic field on thermo-elastic deformations of solid body. As electromagnetic equations are governed by Maxwell equations and elastic field is represented by Hooke's law. But the superposition of these two fields results in the fact that the electro-magnetic field influences the elastic field by entering the elastic stress equations of motion as a body force called as the Lorentz's ponderomotive force, while the elastic field in response to this influences the electro-magnetic field by modifying the Ohm's law. So modified model for this type of media using following Kumar and Rupender (2009a), by taking into account the Lorentz force is:

$$t_{ij} = \lambda u_{k,k}\delta_{ij} + \mu(u_{i,j} + u_{j,i}) + \kappa\,(u_{j,i} - \epsilon_{ijk}\,\phi_k)$$

$$m_{ij} = \alpha\phi_k\delta_{ij} + \beta\phi_{i,j} + \gamma\phi_{j,i}$$

$$(\mu + K)u_{i,jj} + (\lambda + \mu)u_{j,ji} + K\epsilon_{ijk}\phi_{k,j} + \epsilon_{ijk}\,J_j B_k = \rho\ddot{u}_i\,,$$

$$K\epsilon_{ijk}u_{k,j} - 2K\phi_i + (\alpha + \beta)\phi_{j,ji} + \gamma\phi_{i,jj} = \rho j\ddot{\phi}_i\,.$$

Magneto Micropolar thermo elasticity

This theory deals the effects of magnetic field on the elastic deformation produced by uneven heating throughout the body which may or may not be subjected to mechanical forces. In this case, besides the elastic and electro-magnetic fields, thermal field is also present. Each of these fields contributes to the total deformation of the body and interacts with each other. The electro-magnetic field is still governed by Maxwell's equations with, of course, a modified Ohm's law, while the elastic field is determined by the modified Hooke's law and the thermal field by Fourier's law of heat conduction in its modified form. Using Ezzat and Awad (2009) the mathematical model for homogenous isotropic micropolar thermoelastic solid in magnetic field is:

$$t_{ij} = \lambda u_{k,k}\delta_{ij} + \mu(u_{i,j} + u_{j,i}) + \kappa\,(u_{j,i} - \epsilon_{ijk}\,\phi_k) - \vartheta T\delta_{ij}$$

$$m_{ij} = \alpha\phi_k\delta_{ij} + \beta\phi_{i,j} + \gamma\phi_{j,i}$$

$$K^*\nabla^2 T = \left(\frac{\partial}{\partial t} + \tau_o\frac{\partial^2}{\partial t^2}\right)(\rho c^* T + T_o\vartheta e) + \pi_o div\vec{j}$$

$$(\mu + K)u_{i,jj} + (\lambda + \mu)u_{j,ji} + K\epsilon_{ijk}\phi_{k,j} + \epsilon_{ijk}\,J_j B_k - \vartheta T_{,i} = \rho\ddot{u}_i\,,$$

$$K\epsilon_{ijk}u_{k,j} - 2K\phi_i + (\alpha + \beta)\phi_{j,ji} + \gamma\phi_{i,jj} = \rho j\ddot{\phi}_i\,.$$

Review of Literature

(a) Classical theory of elasticity

Classical theory of elasticity is one of the most important branches of continuum mechanics, which deals with the stresses and deformations in elastic solids produced by external forces or change in temperature. Any solid is called elastic when it can undergo deformation under an applied load and can return to its original configuration after release of the deforming loads. An elastic solid that undergoes only an infinitesimal deformation and for which the governing material law is linear is called linear elastic solid. The classical theory of elasticity serves as an excellent model for studying the mechanical behavior of a wide variety of solid materials and is used extensively in civil, mechanical and aeronautical engineering design. This is the oldest established theory governing the behavior of deformable solid materials, founded in its present form in the early nineteenth century. In the theory of linear elasticity, we are concerned with an ideal material governed by Hooke's law, which represents a linear relationship between the stress and strain. Hooke's law has influenced the scientific thoughts for a considerably long period and its results agreed with experiments quite well.

Classical theory is inadequate to represent the behavior of such materials as it assumes that the action across an infinitesimal surface element inside a material is equivalent to a force, whereas in general it is expected that, the action across a hypothetical plane within material should be statically equivalent to a force and couple.

First attempt to remove these discrepancies of the classic, theory of elasticity was made by Voigt (1887) by assuming that the transmission of interaction between two particles of a body through an elementary area lying within the material was affected not solely by the action of a force vector but also by a moment vector. This led to the existence of couple stress. Later Cosserat and Cosserat (1909) gave a unified theory in which every material particle is capable of both linear displacement and rotation during the deformation of the material. Thus, in the Cosserat theory of elasticity, the deformation of the body is described by a displacement vector and an independent rotation vector.

(b) Micropolar theory of elasticity

Eringen (1965) renamed the theory introduced by Eringen and Suhubi (1964) as the micromorphic continuum theory. A micromorphic continuum is a collection of material particles which can undergo classical motion and microrotation. The deformation of material is assumed to be affine. Thus, any material point in this body possesses twelve

degree of freedom: three for macromotion and nine for micronmotion. This theory, in special case contains the Cosserat continuum and the indeterminate couple stress theory. Following these works, an intense activity began and literature now contains several hundred papers in this and in related field.

Eringen (1966) introduced the theory of micropolar elasticity, where a body was assumed consisting of interconnected particles in the form of small rigid bodies undergoing translational motion as well as rotational motion. These elements are allowed to rotate independently. The motion is here described not only by a deformation but also by a microrotation giving six degrees of freedom. Physically, solid propellant grains, polymeric materials and fiberglass come in the category of such materials. The theory is expected to find applications in the treatment of mechanics of granular materials, composites fibrous materials, and particularly microcracks and microfractures.

(c) Micropolar Thermoelasticity

The governing equations of thermoelasticity in the usual frame work of linear coupled thermoelasticity consists of the wave type (hyperbolic) equation of motion and the diffusion type (parabolic) equation of heat conduction. It is seen that a part of the solution of the energy equation tends to infinity. If an isotropic, homogeneous, elastic material is subjected to thermal or mechanical disturbances; the effects in the temperature and displacement field are felt at an infinite distance from the source of disturbance. This implies that a part of the disturbance has an infinite velocity of propagation which is physically impossible. To overcome this drawback, the need was felt to develop theories of generalized thermoelasticity. Two generalizations to the coupled theory were introduced. The first is due to Lord and Shulman (1967), who obtained a wave –type heat equation by postulating a new law of heat conduction to replace the classical Fourier's law. Since the heat equation of this theory is of the wave type, it automatically ensures finite speeds of propagation for heat and elastic waves. The remaining governing equations for this theory, namely, the equations of motion and constitutive relations, remain the same as those for the coupled and the uncoupled theories. The second generalization to the coupled theory of elasticity is what is known as the theory of thermoelasticity with two relaxation times or the theory of temperature – rate- dependent thermoelasticity. The linear theory of micropolar thermoelasticity was developed by extending the theory of micropolar continua to include thermal effects by Nowacki

(1966). Green and Lindsay (1972) obtained an explicit version of the constitutive equations. This theory contains two constants that act as relaxation times and modify all the equations of the coupled theory, not only the heat equation. The classical Fourier's law of heat conduction is not violated if the medium under consideration has a center of symmetry.

Green and Naghdi (1993) proposed the theory of thermoelasticity without energy dissipation and presented the derivation of a complete set of governing equations of the linearized version of the theory for homogeneous and isotropic materials in terms of displacement and temperature fields and proved the uniqueness of the solution of the corresponding initial mixed boundary value problem.

Kumar et al. (2003) investigated source problem in micropolar theory of thermoelasticity and obtained the general solution for a half-space subjected to arbitrary heat source. Kumar and Ailawalia (2005) analyzed deformation in micropolar cubic crystal due to various sources. Hu et al. (2005) proposed a variational method for evaluating the effective behavior of micropolar composites. The influence of reinforcement size on the yielding and strain hardening of particulate composites was investigated. Kumar and Ailawalia (2006b) investigated the response of a micropolar thermoelastic medium possessing cubic symmetry with one relaxation time due to time harmonic series. Kumar and Ailawalia (2006a) studied interactions due to mechanical and thermal sources in a micropolar thermoelastic medium possessing cubic symmetry. A comprehensive study was made by Othman and Singh (2007) in micropolar thermoelastic medium under five theories of generalized thermoelasticity namely Lord-Shulman (L-S) with one relaxation time, Green-Lindsay (G-L) with two relaxation times, Green and Naghdi (G-N) theory without energy dissipation and Chandrasekharaiah-Tzou theory with dual phase lag and coupled theory. Kumar and Ailawalia (2007) studied the steady state response of a micropolar thermoelastic medium without energy dissipation possessing cubic symmetry due to a moving load and concluded that magnitude of moving load velocity on the surface of the solid plays an important role in the deformation of body. Othman et al. (2009) presented a paper on disturbances in a homogenous, isotropic elastic medium with generalized thermoelastic diffusion using G-N theory. Othman *et al.* (2010) with the help of a general model of equations of generalized thermo-microstretch for a homogenous isotropic elastic half-space studied the L-S theory, G-L theory and classical dynamical coupled theory. They used normal mode

analyses to obtain the solution and showed the dependence of modulus of elasticity on reference temperature has a significant effect on the thermal and mechanical interactions.

Features of thermo elastic waves with thermal relaxation in isotropic micropolar plate was investigated by Shaw and Mukhopadhyay (2011) using generalized theory of Lord-Shulman and result found in this study agreed with those predicted by Sharma and Eringen in the context of various theories of classical as well as micropolar thermoelasticity.

(d) Magneto micropolar thermoelasticity

A problem in generalized magneto thermoelasticity in a conducting medium with variable material properties was also studied by Youssef (2006). Kumar and Rupender (2009a) studied a two dimensional problem in electromagnetic micropolar generalized thermoelastic medium in the presence of a transverse magnetic field subjected to mechanical force or thermal source when entire medium is rotating with uniform angular velocity and observed that the application of thermal source affects more in comparison to the mechanical force. Kumar and Rupender (2009b,c) investigated a problem in electromagnetic micropolar elasticity in two dimensional half space subjected to distributed thermal and mechanical sources in the presence of transvers magnetic field and found that for both S-L and G-N theory of thermoelasticity produce similar variations in the behaviors of normal strain, normal stress, tangential couple stress and temperature distribution. Ezzat and Bary (2009) analyzed a one dimensional model of two-temperature generalized magneto-thermoelastic theories with one relaxation time in perfectly conducting medium using state space approach subjected to a thermal shock and traction-free surface and indicated that two-temperature generalized theory of magneto-thermo-viscoelascticity describes the behavior of the particles of an elastic body more realistically than the one-temperature theory. Ezzat and Awad (2009) introduced the modified Ohm's law, including the temperature gradient and charge density effects, and the generalized Fourier's law including current density effect to the equations of the linear theory of micropolar generalized magneto thermoelasticity. Normal mode analysis is used to obtain the solution. He and Cao (2009) used generalized thermoelastic theory with thermal relaxation in context of L-S theory to investigate the magneto-thermoelastic problem of thin slim strip placed in a magnetic field and subjected to a moving plane of heat source and found that magnetic field significantly influences the variations of non-

dimensional displacement and stress but has no effect on the non-dimensional temperature.

Objective:

The aim of current study is to formulate a mathematical model and solve a two dimensional problem in magneto micropolar thermoelastic medium by using either Integral transform approach or Eigen value approach. Plane strain problems and axisymmetric problems in the medium will be discussed due to its increasing importance in engineering applications. Also a comparison will be done by removing coupling coefficients, to see the effect of applied load (mechanical or thermal) in the absence of either magnetic field or thermal effect or micropolar effect. Response of suitable crystal (Al or Mg) will be analyzed under the action of mechanical, thermal and magnetic forces by applying Lord-Shulman (L-S), Green-Lindsay (G-L) and Green-Naghdi (G-N) theories using the elastic values of these as given in Eringen (1966) and Gaüthier (1982). For verification results obtained with the undertaken model will be compared with the existing models as obtained by some of the authors Kumar and Rupender (2009a), Ezzat and Awad (2009), Kumar and Singh (2002) etc. The study undertaken will be limited to mathematical modeling and solution, but in future, there is possibility that work can be extended to experimental verification.

Research methodology

The formulae of temperature distribution, displacements, micro-rotation, stresses, couple stresses, electric field, magnetic field and current density will be obtained under the applied load. After getting the solution in the physical domain the components of stress, strain, temperature distribution, magnetic field and electric field will be analyzed graphically with the help of suitable crystal Al or Mg. In current study the sources will be linearly or uniformly distributed. After using the appropriate model, the set of partial differential equations will be solved by using following two techniques:

(a) Integral transforms

Laplace, Fourier and Hankel transform come under this category which may be defined as:

$$\overline{f}(x, z, p) = \int_0^\infty f(x, z, t)\, exp(-pt)dt\,,$$

$$\widetilde{f}(\xi, z, p) = \int_{-\infty}^{\infty} \overline{f}(x, z, p)\, e(-\iota\xi\, x)dx\,,$$

$\tilde{f}(\xi, z, p) = \int_0^\infty \bar{f}(r, z, p) r J_n(\xi r)dr$ respectively.

For plane strain problems Laplace and Fourier transforms and for axisymmetric problems Laplace and Hankel transforms will be used respectively to convert differential coefficients into equivalent algebraic terms and using these transformations, set of partial equations will be transformed into frequency domain and after obtaining solution of these set of equations in frequency domain they will be inverted into physical domain using inverse numerical techniques.

Eigen Value approach

Using mathematical model of the undertaken problem, set of differential equations obtained will be written in vector matrix form as

$$\frac{d}{dz} W(\xi, z, p) = A(\xi, p) W(\xi, z, p)$$

where

$$W = \begin{bmatrix} U \\ DU \end{bmatrix},\ U = \begin{bmatrix} \tilde{u}_1 \\ \tilde{u}_3 \\ \tilde{\varphi}_2 \\ \tilde{T} \end{bmatrix},$$

Typical solution of above differential equation can be taken as

$W(\xi, z, p) = X(\xi, p)\ e^{q z}$, for some q

So we obtain $A(\xi, p)W(\xi, z, p) = qW(\xi, z, p),$ which leads to eigen value problem. Advantage of using eigen value approach over the earlier used static methods is that it avoids integration by means of matrix exponential thus simplifying expressions to a great extent.

List of publications

1. Singh R. and Kumar V. (2011): *Thermo-Mechanical deformation in magneto micropolar thermoelastic medium with modified Fourier and Ohm's law*. - International Journal of Applied Mechanics and Engineering, vol.16, pp.83-105.
2. Singh R., Singh K. and Kumar V. (2011): *Response due to mechanical and thermal sources in micropolar generalized thermoelastic medium*. -International Journal of Applied Mechanics and Engineering, vol.16, pp.107-128.

Paper published in conference/seminar proceedings

A paper with title "Plane strain problem in micropolar generalized thermoelastic solid" was presented and published in UGC sponsored National seminar in November 2010.

Nomenclature

λ, μ	= Lame's constants
α, β ,γ ,K	= Micropolar material constants
ρ	= Density
j	= Micro-inertia
u	= Displacement vector
Φ	= Microrotation vector
t_{ij}	= Force stress tensor
m_{ij}	= Couple stress tensor
λ_l	= Microstress tensor
δ_{ij}	= Kronecker delta
ε_{ijk}	= Alternating tensor
Δ	= Gradient operator
ι	= Iota

And dot denotes the partial derivative w.r.t. time.

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